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# On realizing the shortest time strategy in a CA FF pedestrian dynamics model

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A mathematical model of pedestrian movement on the basis of the stochastic cellular automata approach is proposed. A floor field (FF) model is taken as a basis model. The FF models imply that virtual people follow the shortest path strategy; meanwhile, they also follow the shortest time strategy. The focus of this study is mathematical formalization and model implementation of these features of pedestrian movement. Some results of the simulation are reported.

## 1 Introduction

This study is aimed at the mathematical simulation of pedestrian movement. We will consider a stochastic cellular automata (CA) model. Numerous collective (macroscopic) properties, such as the lane formation, oscillations of the direction in bottlenecks, the “faster-is-slower” effect are reproduced by the currently existing models of pedestrian movement, which form a basis for

the pedestrian modelling. However, in the microscopic aspect, there are still things to be done. The better the behavior of an individual pedestrian in the model, the more realistic the collective interaction and the flow shape. The aim of this study is to formalize mathematically and implement in the model the behavioral aspects of the decision making process, considering that moving people follow at least two strategies, which are the shortest path and the shortest time. Depending on a current position, the strategies may vary, cooperate, and compete. According to the comprehensive theory of the pedestrian dynamics [Schadschneider et. al. 2009, Helbing 2001], such a model can be considered as a tactical level when the pedestrians make short-term decisions.

The model imports idea of a map from the floor field (FF) CA model [Nishinari et.al. 2004]. The map provides pedestrians with the information on ways to the exits. In our model, the map depicts the shortest distance from the current position to a target.

The article is organized as follows. In the next section, the problem is stated. In Section 3, we describe the model in general and update rules. Section 3.2 contains probability formulas. It is followed by the discussion and the presentation of the simulation results.

## 2 Statement of the problem

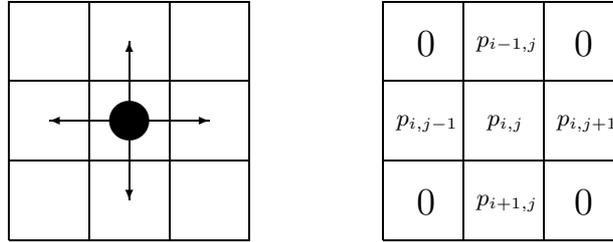
The space (plane) is known and sampled into cells  $40cm \times 40cm$  in size which are either empty or occupied by the only pedestrian (particle) [Nishinari et.al. 2004]. The cells can contain walls and other stationary obstacles. Thus, the space is presented by two matrixes:

$$f_{ij} = \begin{cases} 1, & \text{cell } (i, j) \text{ is occupied by a pedestrian;} \\ 0, & \text{cell } (i, j) \text{ is empty,} \end{cases}$$

$$w_{ij} = \begin{cases} 1, & \text{cell } (i, j) \text{ is occupied by an obstacle;} \\ 0, & \text{cell } (i, j) \text{ is empty.} \end{cases}$$

In the model, static floor field  $S$  is used. Field  $S$  coincides with the sampled space. Each  $S_{i,j}$  value stores the information on the shortest distance from cell  $(i, j)$  to the nearest exit; i.e., field  $S$  increases radially from the exit cells. Field  $S$  of the exit cells is zero. It does not change with time and is independent of the presence of the particles. Field  $S$  can be considered as a map used by the pedestrians during their movement to the nearest exit.

The starting positions of people are known. A target point of each pedestrian is the nearest exit. Each particle can move to one of its four adjacent cells or stay in the present cell (the von Neumann neighborhood) at each discrete time step  $t \rightarrow t + 1$  — fig. 1; i.e.,  $v_{max} = 1[step]$ .



**Fig. 1.** Target cells for a pedestrian in the next time step.

Generally speaking, the direction of the movement of each particle at each time step is random and determined in accordance with the distribution of transition probabilities and transition rules.

Thus, the main problem to solve is determination of the “right” transition probabilities and transition rules.

### 3 Solution

#### 3.1 Update rules

We will use a scheme typical of the stochastic CA models. At the first stage, some preliminary calculations are made. Then, at each time step the transi-

tion probabilities are calculated, and the directions are selected. In the case, when there are more than one candidate to occupy a cell, a conflict resolution procedure is applied. Finally, a simultaneous transition of all the particles is made.

In our case, the *preliminary step* includes the calculation of FF  $S$ . Each cell  $S_{i,j}$  stores the information on the shortest discrete distance to the nearest exit. The unit of this distance is a number of steps. In order to calculate field  $S$  (for this purpose only), we admit diagonal transitions. The length of the vertical and horizontal path to the nearest cell is 1 and that of the diagonal path is  $\sqrt{2}$ . (Since movement through walls or columns is impossible, in these cases roundabout movement is assumed.) This method makes a discrete distance close to the continuous one.

The probabilities of movement from cell  $(i, j)$  to each of the four adjacent cells are calculated as

$$p_{i-1,j} = \frac{\tilde{p}_{i-1,j}}{N_{i,j}}, p_{i,j+1} = \frac{\tilde{p}_{i,j+1}}{N_{i,j}}, p_{i+1,j} = \frac{\tilde{p}_{i+1,j}}{N_{i,j}}, p_{i,j-1} = \frac{\tilde{p}_{i,j-1}}{N_{i,j}}, \quad (1)$$

where  $N_{i,j} = \tilde{p}_{i-1,j} + \tilde{p}_{i,j+1} + \tilde{p}_{i+1,j} + \tilde{p}_{i,j-1}$ .

Moreover

$$p_{i-1,j} = 0, \quad p_{i,j+1} = 0, \quad p_{i+1,j} = 0, \quad p_{i,j-1} = 0 \quad (2)$$

only if

$$w_{i-1,j} = 1, \quad w_{i,j+1} = 1, \quad w_{i+1,j} = 1, \quad w_{i,j-1} = 1 \quad (3)$$

respectively.

The probability of retaining the current position is not calculated directly. Nevertheless, the decision rules are organized so that such opportunity could be taken; thus, peoples' patience is implemented.

**The decisions rules** are the following [Kirik et al 2007]:

1. If  $N_{i,j} = 0$ , motion is forbidden; otherwise, target cell  $(l, m)^*$  ( $l \in \{i - 1, i, i + 1\}$ ,  $m \in \{j - 1, j, j + 1\}$ ) is chosen randomly using the transition probabilities.

2. a) If  $N_{i,j} \neq 0$  and  $(1 - f_{l,m}^*) = 1$ , then target cell  $(l, m)^*$  is fixed.
- b) If  $N_{i,j} \neq 0$  and  $(1 - f_{l,m}^*) = 0$ , then the cell  $(l, m)^*$  is not available as it is occupied by a particle, and a “peoples’ patience” can be implemented. For this purpose, the probabilities of cell  $(l, m)^*$  and all the other occupied adjacent cells are given for the current position<sup>4</sup>. Again, the target cell is chosen randomly using the transformed probability distribution<sup>5</sup>.
3. Whenever two or more pedestrians have the same target cell, movement of all the involved pedestrians is denied with probability  $\mu$ ; i.e., all the pedestrians stay at their current positions [Nishinari et.al. 2004]. One of the candidates moves to the desired cell with the probability  $1 - \mu$ . The pedestrian allowed to move is chosen randomly<sup>6</sup>.
4. The pedestrians that are allowed to move perform their motion to the target cell.
5. The pedestrians that appear in the exit cells leave the room.

<sup>4</sup> For example, the probability distribution is

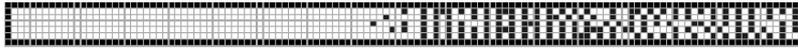
$\alpha$	N	S	W	E	C
$p$	$p_{i-1,j}$	$p_{i+1,j}$	$p_{i,j-1}$	$p_{i,j+1}$	0

where  $\alpha$  (random variable) is the direction.  $N_{i,j} \neq 0$ , and let target cell  $(l, m)^*$  is  $(i - 1, j)^*$ . Let  $(1 - f_{i-1,j}^*) = 0$  and  $(1 - f_{i,j-1}) = 0$ . Then cells  $(i - 1, j)^*$  and  $(i, j - 1)$  are occupied by pedestrians, and directions  $N$  and  $W$  are forbidden for movement. Thus, the probability distribution takes the form:

$\alpha$	N	S	W	E	C
$p_{ij}$	0	$p_{i+1,j}$	0	$p_{i,j+1}$	$p_{i-1,j}^* + p_{i,j-1}$

<sup>5</sup> This trick of choosing the current position is provoked by the fact that when moving directionally people usually stop only if the preferable direction is occupied. The original FF model [Schadschneider et. al. 2009] never gives zero probability to the current position, and it may be chosen independent of the environment.

<sup>6</sup> A technique for adaptation of parameter  $\mu$  to a rectangular room with one exit is described in [Kirik et al 2007]. There is also the advanced method [Yanagisawa et. al. 2007]. The exit problem is beyond our consideration; we only need to provide the flow pass through narrow places.



**Fig. 2.** The shortest path strategy evacuation in the simple space.

The above rules are applied to all the particles at the same time; i.e., parallel update is used.

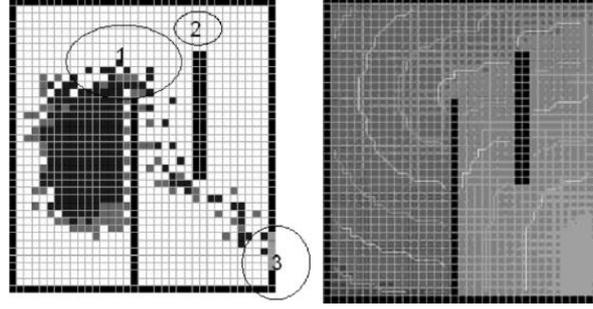
### 3.2 Probability calculation

In this study, we focus mainly on the transition probabilities. In normal situations people choose their route carefully (see [Helbing 2001] and reference therein). The pedestrians keep themselves at a certain distance from other people and obstacles. The tighter the crowd and the more in a hurry a pedestrian, the smaller this distance. During movement, people follow at least two strategies: the shortest path and the shortest time.

In the models using the static floor field, the wish of a particle to move to the nearest exit is independent of the current distance to it. The main driving force making a pedestrian move to the exit is to minimize  $S$  at each time step<sup>7</sup>. From the mathematical view point, this means that among all the nearest cells the neighbor with the smallest  $S$  has the highest transition probability. However, in this case, only the shortest path strategy is mainly implemented and a slight regard to avoid congestions is supposed, so this does not seem realistic for peoples' movement.

It's necessary to note that for simple spaces (see Fig. 2) it's not dramatic because two strategies coincide, and static-floor-field-based-models may give appropriate results: simulation seems realistic and a dependence between density and velocity (fundamental diagram) is qualitatively good. But for the same model in a more complex space a slight regard to an avoidance of congestions is supposed. In Fig.3a a typical example of such behavior is presented: a small(unrealistic) turn radius (point 1) that results in congestion before turn,

<sup>7</sup> If field  $S$  decreases from the exit, then, the main driving force is maximizing  $S$ .



a) The shortest path strategy evacuation for complex space.      b) Field  $S$ .

**Fig. 3.**

detours facilities are not used (point 2), an ineffective use of the exit width (point 3).

In order to improve the dynamics in the FF model, an environmental analyzer is proposed to be introduced to the probability formula. This should reduce the effect of the short path strategy and increase the probability to move in the direction favorable to movement. Thus, a kind of “trade off” between the two main strategies would be achieved.

We introduce a revised version of the environmental analyzer [Kirik et al 2007] and attempt to formalize mathematically the decision making process which is fairly complex as people choose their paths while the strategies of their movement vary (cooperate, coincide, and compete) depending on the current position and the environment; in other words, on place and time.

First, we present the transition probability formula; below, we will discuss it in detail. For example, the probability of movement from cell  $(i, j)$  to the upper neighbor is

$$p_{i-1,j} = N_{i,j}^{-1} A_{i-1,j}^{SFF} A_{i-1,j}^{people} A_{i-1,j}^{wall} (1 - w_{i-1,j}). \quad (4)$$

Here

- $A_{i-1,j}^{SFF} = \exp(k_S \Delta S_{i-1,j})$  is the main driven force:

1.  $\Delta S_{i-1,j} = S_{i,j} - S_{i-1,j}$ ;
2.  $k_S \geq 0$  is the (model) sensitivity parameter which can be interpreted as knowledge of the shortest way to the destination point or a wish to move to the destination point. The equality  $k_S = 0$  means that the pedestrians ignore the information from field  $S$  and move randomly. The higher  $k_S$ , the better directed the movement.

Since field  $S$  increases radially from the exit(s) in our model, then,  $\Delta S_{i-1,j} > 0$  if cell  $(i-1, j)$  is closer to the exit than the current cell  $(i, j)$ ,  $\Delta S_{i-1,j} < 0$  if the current cell is closer, and  $\Delta S_{i-1,j} = 0$  if cells  $(i, j)$  and  $(i-1, j)$  are equidistant from the exit.

Originally pure values of field  $S$  are used in the probability formula in FF models, e.g. [Nishinari et.al. 2004, Henein et. al. 2007, Kretz et.al., Yanagisawa et. al. 2007, Schadschneider et. al. 2009]. We propose to use only a value of gradient  $\Delta S_{i-1,j}$  [Kirik et. al. 2009]. From a mathematical view point, it yields the same result<sup>8</sup> but computationally this trick has a great advantage. The values of field  $S$  may be too high (it depends on the space size); in this case,  $\exp(k_S S_{i-1,j})$  can appear uncomputable. This is a significant limitation of the models. At the same time,  $0 \leq \Delta S_{i-1,j} \leq 1$ , and the computation of  $A_{i-1,j}^{SFF}$  does not make difficulties;

<sup>8</sup> The main driven force term is the only term which affects the inference; the rest terms in the probability formula are omitted here. Let field  $S$  increases towards the exit as in the original FF model; then,  $\Delta S_{i-1,j} = S_{i-1,j} - S_{i,j}$ ,  $\Delta S_{i,j+1} = S_{i,j+1} - S_{i,j}$ ,  $\Delta S_{i+1,j} = S_{i+1,j} - S_{i,j}$ ,  $\Delta S_{i,j-1} = S_{i,j-1} - S_{i,j}$ , and  $S_{i-1,j} = \Delta S_{i-1,j} + S_{i,j}$ ,  $S_{i,j+1} = \Delta S_{i,j+1} + S_{i,j}$ ,  $S_{i+1,j} = \Delta S_{i+1,j} + S_{i,j}$ ,  $S_{i,j-1} = \Delta S_{i,j-1} + S_{i,j}$ . Therefore, we have  $\frac{e^{k_S S_{i-1,j}}}{e^{k_S S_{i-1,j}} + e^{k_S S_{i,j+1}} + e^{k_S S_{i+1,j}} + e^{k_S S_{i,j-1}}} = \frac{e^{k_S S_{i,j}} e^{k_S \Delta S_{i-1,j}}}{e^{k_S S_{i,j}} (e^{k_S \Delta S_{i-1,j}} + e^{k_S \Delta S_{i,j+1}} + e^{k_S \Delta S_{i+1,j}} + e^{k_S \Delta S_{i,j-1}})} = \frac{e^{k_S \Delta S_{i-1,j}}}{e^{k_S \Delta S_{i-1,j}} + e^{k_S \Delta S_{i,j+1}} + e^{k_S \Delta S_{i+1,j}} + e^{k_S \Delta S_{i,j-1}}}$ . If field  $S$  increases from the exit as in our model, the gradients are calculated in the opposite direction; i.e.,  $\Delta S_{i-1,j} = S_{i,j} - S_{i-1,j}$ ,  $\Delta S_{i,j+1} = S_{i,j} - S_{i,j+1}$ ,  $\Delta S_{i+1,j} = S_{i,j} - S_{i+1,j}$ ,  $\Delta S_{i,j-1} = S_{i,j} - S_{i,j-1}$ , and field  $S$  has to have negative sign.

- $A_{i-1,j}^{people} = \exp(-k_P D_{i-1,j}(r_{i-1,j}^*))$  is the factor taking into account people density in the given direction:
  1.  $r_{i-1,j}^*$  is the distance to the nearest obstacle in the given direction ( $r_{i-1,j}^* \leq r$ );
  2.  $r > 0$  is the visibility radius (model parameter) representing the maximum distance (number of cells) at which the people density and the presence of obstacles influence on the probability in the given direction;
  3. the density lies within  $0 \leq D_{i-1,j}(r_{i-1,j}^*) \leq 1$ ; if all the  $r_{i-1,j}^*$  cells are empty in this direction, we have  $D_{i-1,j}(r_{i-1,j}^*) = 0$ ; if all the  $r_{i-1,j}^*$  cells are occupied by people in this direction, we have  $D_{i-1,j}(r_{i-1,j}^*) = 1$ . We estimate the density using the idea of the kernel Rosenblatt-Parzen density estimate ([Rosenblatt 1956], [Parzen 1956]):

$$D_{i-1,j}(r_{i-1,j}^*) = \frac{\sum_{m=1}^{r_{i-1,j}^*} \Phi\left(\frac{m}{C(r_{i-1,j}^*)}\right) f_{i-m,j}}{r_{i-1,j}^*},$$

were

$$\Phi(z) = \begin{cases} (0.335 - 0.067(z)^2) 4.4742, & |z| \leq \sqrt{5}; \\ 0, & |z| > \sqrt{5}, \end{cases} \quad (5)$$

$$C(r_{i-1,j}^*) = \frac{r_{i-1,j}^* + 1}{\sqrt{5}};$$

4.  $k_P$  is the (model) people sensitivity parameter which determines the effect of the people density. The higher parameter  $k_P$ , the more pronounced the shortest time strategy;
- $A_{i-1,j}^{wall} = \exp\left(-k_W \left(1 - \frac{r_{i-1,j}^*}{r}\right) \tilde{1}(\Delta S_{i-1,j} - \max \Delta S_{i,j})\right)$  is the factor taking into account walls and obstacles:
    1.  $k_W \geq k_S$  is the (model) wall sensitivity parameter which determines the effect of walls and obstacles;
    2.  $\max \Delta S_{i,j} = \max\{\Delta S_{i-1,j}, \Delta S_{i,j+1}, \Delta S_{i+1,j}, \Delta S_{i,j-1}\}$ ,

$$\tilde{1}(\phi) = \begin{cases} 0, & \phi < 0, \\ 1 & \text{otherwise.} \end{cases}$$

An idea of the function  $\tilde{1}(\Delta S_{i-1,j} - \max \Delta S_{i,j})$  comes from the fact that people avoid obstacles only when moving towards the destination point.

When people make detours (in this case, field  $S$  is not minimized), approaching the obstacles is not excluded.

- NOTE that only walls and obstacles turn the transition probability to “zero”.

The probabilities of movement from cell  $(i, j)$  to each of the four neighbors are:

$$p_{i-1,j} = N_{i,j}^{-1} \exp[k_S \Delta S_{i-1,j} - k_P D_{i-1,j}(r_{i-1,j}^*) - k_W (1 - \frac{r_{i-1,j}^*}{r}) \tilde{1}(\Delta S_{i-1,j} - \max \Delta S_{i,j})] (1 - w_{i-1,j}); \quad (6)$$

$$p_{i,j+1} = N_{i,j}^{-1} \exp[k_S \Delta S_{i,j+1} - k_P D_{i,j+1}(r_{i,j+1}^*) - k_W (1 - \frac{r_{i,j+1}^*}{r}) \tilde{1}(\Delta S_{i,j+1} - \max \Delta S_{i,j})] (1 - w_{i,j+1}); \quad (7)$$

$$p_{i+1,j} = N_{i,j}^{-1} \exp[k_S \Delta S_{i+1,j} - k_P D_{i+1,j}(r_{i+1,j}^*) - k_W (1 - \frac{r_{i+1,j}^*}{r}) \tilde{1}(\Delta S_{i+1,j} - \max \Delta S_{i,j})] (1 - w_{i+1,j}); \quad (8)$$

$$p_{i,j-1} = N_{i,j}^{-1} \exp[k_S \Delta S_{i,j-1} - k_P D_{i,j-1}(r_{i,j-1}^*) - k_W (1 - \frac{r_{i,j-1}^*}{r}) \tilde{1}(\Delta S_{i,j-1} - \max \Delta S_{i,j})] (1 - w_{i,j-1}); \quad (9)$$

### 3.3 Discussion

In expressions (6)-(9), the product  $A^{people} A^{wall}$  is the environmental analyzer that deals with people and walls. The following restrictions take place  $0 \leq \Delta S \leq 1$ ,  $0 \leq D(r^*) \leq 1$ , and  $0 \leq 1 - \frac{r^*}{r} \leq 1$ . These allows adjusting

sensitivity of the model to the people density and the approaching to obstacles using parameters  $k_P$  and  $k_W$ , respectively. To be pronounced people and wall terms should not have parameters less than  $k_S$  ( $k_P \geq k_S$ ,  $k_W \geq k_S$ ). Term  $A^{wall}$  corresponds only to avoidance of the ahead obstacles so it is not discussed here. Assume that  $k_W = k_S$ .

Following the shortest time strategy implies that, wherever possible, the detours around high-density regions are made. Term  $A^{people}$  reduces the main driving force, which provides following the shortest path strategy, and the probability of the detours grows. The higher  $k_P \geq k_S$ , the more pronounced the shortest time strategy. Of course,  $k_P$  can not increase infinitely, and we propose the following upper bound  $k_P \leq 4k_S$ .

Note that probabilities are density adaptive; the low people density lowers the effect of  $A^{people}$ , and the probability of the shortest path strategy increases automatically.

### 3.4 Analogies

Having referred to [Schadschneider et. al. 2009], one could interpret the term  $A^{people} A^{wall}$  as an element of matrix of preferences  $M$ .

The idea closest to ours is implemented in the F.A.S.T.-model [Kretz et.al.], which is also based on the FF model. The probability formula contains the same term  $A^{people}$ , e.g.  $A_{i-1,j}^{people} = \exp(-k_P D_{i-1,j})$ , with parameter  $k_P$  of the people density sensitivity, but a number of particles in the Moore neighborhood of the destination cell under consideration is used as density  $D$  ( $0 \leq D \leq 8$ <sup>9</sup>). This density estimate is independent on parameter  $r$  (visibility radius parameter) and simplifies the computation. At the same time, in the F.A.S.T.-model pure values of field  $S$  are used; i.e.,  $0 \leq S_{i,j} < \infty$ . As we mentioned above, reasonable reduction of the main driving force requires a comparable magnitude of the people and field  $S$  terms. As a result, parameter

<sup>9</sup> The F.A.S.T.-model assumes  $v_{max} \geq 1$ .

$0 < k_P(i, j) < \infty$  becomes nonconstant (space-dependent). The bigger space, the more pronounced this dependence.

## 4 Simulations

In this section, we present some results of the simulation. The first goal is to demonstrate that our idea works. Then, an influence of the visibility radius parameter and the density sensitive parameter to model dynamics is investigated. The Macromedia Flash 8 was used to make our computer program.

### 4.1 Case study

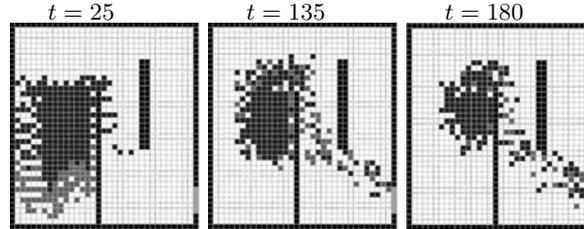
For all experiments the space is  $14.8m \times 13.2m$  ( $37 \text{ cells} \times 33 \text{ cells}$ ) in size and has one exit ( $2.0m$ ). It is sampled into the cells  $40cm \times 40cm$  in size which are either empty or occupied by one pedestrian only (exclusive principle). Static field  $S$  is presented in Fig. 3b. Particles move towards exit with the velocity  $v = v_{max} = 1$ . The high initial density ( $> 3.0[1/m^2]$ ) example is interesting because only in this case unrealistic shapes of flow begin to be a well pronounced under movement in according to the shortest path strategy only.

### 4.2 Difference of the flow dynamics

To demonstrate a quality difference of the flow dynamics two sets of the parameters of the model are used.

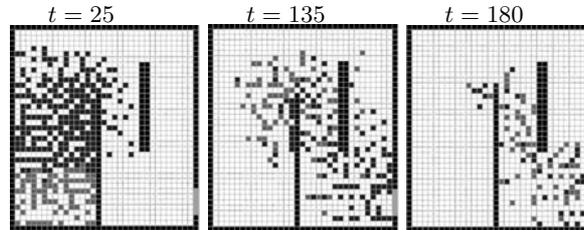
The first set of parameters is  $k_S = k_W = 4$ ,  $k_P = 6$ ,  $r = 10$ . The second set is  $k_S = k_W = 4$ ,  $k_P = 18$ ,  $r = 10$ . Both sets reproduce the following conditions of the movement: the way to the exit is well-known for the pedestrians, the pedestrians want to reach the exit (parameter  $k_S$  is responsible for these conditions); visibility is good ( $r$ ); and the attitude to walls is "loyal" ( $k_W = k_S$ ). The only parameter  $k_P$  is variable.

In the first case ( $k_P = 6$ ), the shortest path strategy predominates. Fig. 4 presents screenshots of the evacuation in different moments. One can see the permanent ineffective small turn radius that results in congestion before turn; detours facilities are not used.



**Fig. 4.** Evacuation for 300 people,  $k_S = k_W = 4$ ,  $r = 10$ ,  $k_P = 6$ ;  $T_{tot} = 320[step]$ .

The other set of the parameters  $k_S = k_W = 4$ ,  $k_P = 18$ , and  $r = 10$  (Fig. 5) makes it possible to realize both strategies depending on the conditions. Recall that term  $A^{people}$  in (6)-(9) works only if  $D(r^*) > 0$  and reduces the transition probability depending on the value of the density in the given direction.



**Fig. 5.** Evacuation for 300 people,  $k_S = k_W = 4$ ,  $r = 10$ ,  $k_P = 18$ ;  $T_{tot} = 270[step]$ .

If the shortest path direction has a high density,  $D(r^*) \approx 1$ , the probability of this direction goes down. At the same time, the probability of direction(s) that are more favorable to movement ( $D(r^*) \ll 1$ ) rises, and the detours around high-density regions are made. One can see the higher turn radius, the

using of detours facilities, the effective use of the exit width, and more realistic shape of flow in a whole.

Thus, under equal movement conditions ( $k_S = k_W = 4$ ,  $r = 10$ ) different density sensitive parameters give significant divergence in the dynamics of the model. Combining of the shortest path and the shortest time strategies gives faster evacuation process,  $T_{tot} = 270[step]$ , then the case when the shortest path strategy predominates,  $T_{tot} = 320[step]$ .

### 4.3 The influence of $k_P$

The influence of density sensitive parameter  $k_P$  to the model dynamics is shown in this subsection. In the first set of experiments, see Fig. 6, visibility radius is fixed,  $r = 10$ , and density sensitive parameter  $k_P$  varies from 2 to 30.

Screenshots in Fig.6 show different shapes of flows depending on the values of parameter  $k_P$ . A very tight congestion before the turn corresponds low values of  $k_P$ . In these cases the influence of density term is not (or very miserable) pronounced. As a result, for the particles from the front part of the flow among all not occupied candidates a position that is the closest to the exit has the highest probability. Preferably this position is a current place of the particle, and collective dynamics in this case is: the particles mainly not to leave present position until the shortest path will free.

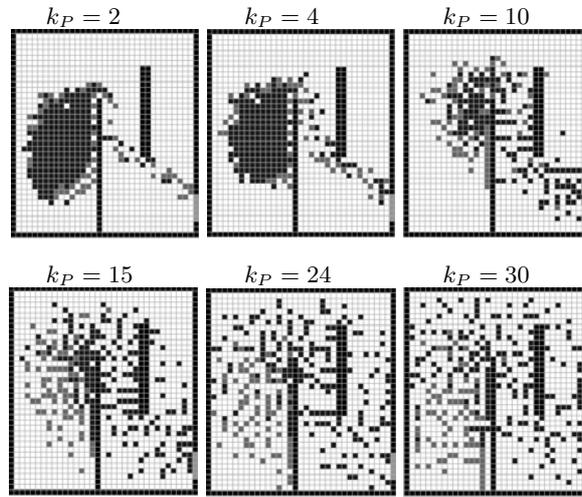
Increasing  $k_P$  implements the possibility of the shortest time strategy. For the particles from the front part of the flow the density term increases probability of the candidate that has more favorable condition to movement. Speaking about the turn region, such direction is opposite to decreasing of field  $S$ .

As it was said above, increasing  $k_P$  should have reasonable limit. A diamond line in Fig. 8 shows that total evacuation time is very  $k_P$  dependant. Minimal value,  $T_{tot} = 260[steps]$ , gives  $k_P = 15$ . This couple of parameters, correspond to the situation when particles know about the exit, want go to

the exit, see the surrounding environment ( $k_S = k_W = 4, r = 10$ ), use the time effective strategy ( $k_P = 15$ ).

The shortest time strategy (when values of  $k_P$  are low) is the most time consuming.

High values of  $k_P$  gives very many local fluctuations of the particles. Such behavior may be interpreted as particles want to go to the exit but afraid to approach to each other, e.g., in a case of an epidemic. Fig. 8 shows that total number of movement in all direction (including not to leave present position) are approximately equal. As a result total evacuation time goes up.



**Fig. 6.** Screenshots of evacuations for 300 people at  $t = 100$  for some  $k_P$  and  $k_S = k_W = 4, r = 10$ .

#### 4.4 The influence of $r$

The influence of visibility radius parameter  $r$  to the model dynamics is shown in this subsection. The density sensitive parameter is fixed,  $k_P = 15$ , and radius  $r$  varies from 1 to 30 (Fig. 9).

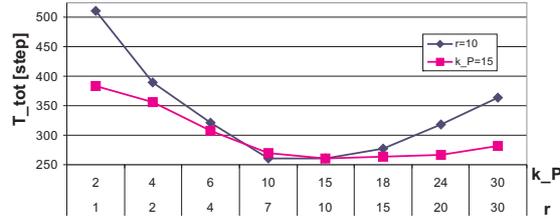


Fig. 7. Total evacuation time  $T_{tot}$  [steps] for two experiments.

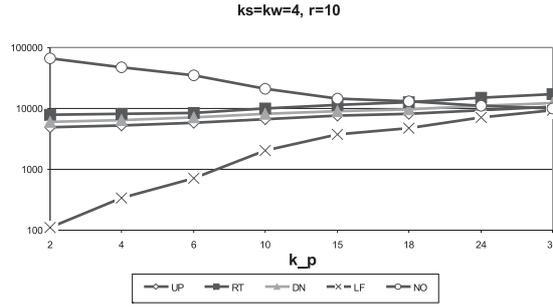
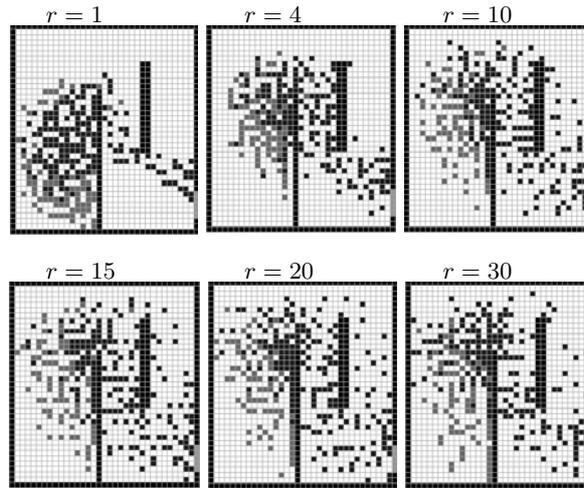
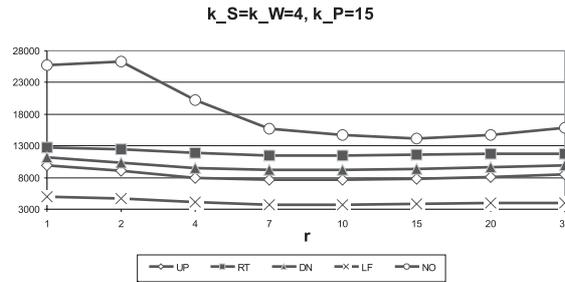


Fig. 8. Total number of steps in each direction for some  $k_P$  and  $k_S = k_W = 4$ ,  $r = 10$ .

A square-line in Fig. 7 shows that total evacuation time is wall sensitive parameter  $r$  dependant under low values of  $r$ . The higher  $r$  the less pronounced dependence between  $r$  and total evacuation time. In these series of experiments  $k_P = 15$ ; particles use the time effective strategy. Value of  $r$  depicts the visibility situation. If  $r = 1$  people move by touch (e.g., in a dark or in a fog or in a heavy smoke). Increasing of  $r$  corresponds to improving of visibility conditions. As in the real life in our simulation visibility conditions reach a point ( $r = 15$ ) when they not restrict (not influence on) the decision making process and the movement of particles. This statement is supported by Fig. 10 which shows that numbers of movement in each direction are approximately equal starting with  $r = 10$ .



**Fig. 9.** Screenshots of evacuations for 300 people at  $t = 100$  for some  $r$  and  $k_S = k_W = 4$ ,  $k_P = 15$ .



**Fig. 10.** Total number of steps in each direction for some  $r$  and  $k_S = k_W = 4$ ,  $k_P = 15$ .

## 5 Conclusion

Implementing the possibility to move in accordance to the shortest time strategy makes more realistic the shape of flow. Figures 4-5 show a great difference in the flow dynamics that obtained by following only one movement strategy and by “keeping in mind” both strategies at a time. It is shown that total evacuation time is people density sensitive parameter  $k_P$  dependent. The low-

est value of  $T_{tot}$  corresponds to the most realistic dynamics. Dependence on visibility conditions (parameter  $r$ ) in the simulations is similar to real life phenomena. The further model dynamics investigation is going on. A necessity of the  $k_P$  spatial adaptation is already obvious.

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